1 Integers

INTRODUCTION

You have learnt about natural numbers, whole numbers and integers in class VI. You have studied that counting numbers 1, 2, 3, 4, 5 ... are called *natural numbers*. The set of natural numbers is denoted by **N**.

Thus, $N = \{1, 2, 3, 4, 5, ...\}$

The set of *whole numbers* contains the counting numbers 1, 2, 3, 4, 5, ... and the number 0. The whole numbers set is denoted by **W** and expressed as

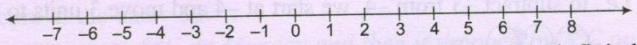
$W = \{0, 1, 2, 3, 4, 5, ...\}$

The set of *Integers* contains whole numbers and negatives of counting numbers. The set of integers is often denoted by **Z**.

Thus, $Z = \{... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$

Note that the numbers 1, 2, 3, 4, ... (and so on) greater than zero (0) are called **positive integers**. These numbers are to the right of zero on the number line.

The numbers ...., –3, –2, –1, which are negatives of counting numbers 1, 2, 3, ... are less than zero and called **negative integers**. These numbers are to the left of zero on the number line. All negative integers are less than 0 and also less than all positive integers.



(These are all integers and they continue left and right infinitely.)

The integers zero (0) is neither positive nor negative.

The numbers 1 and –1, 2 and –2, 3 and –3, 4 and –4 etc. are called **opposites**.

For example, opposite of 85 is –85, opposite of 147 is –147 and opposite of 263 is –263.

These are also called *additive inverse* of each other.

Remember

1. Integers = {...,–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}.
2. Negative integers = {..., –5, –4, –3, –2, –1}.
3. Positive integers = {1, 2, 3, 4, 5, ...}.
4. Zero (0) is neither positive nor negative.

Absolute value of an integer: The absolute value of an integer is the numerical value without regard to whether the sign is negative or positive. The symbol for absolute value is to enclose the number between vertical bars such as | –9 |, | 16 |, etc.

Examples: | –9 | = 9, | 16 | = 16, | –25 | = 25, | –81 | = 81, | 74 | = 74.

ADDITION AND SUBTRACTION OF INTEGERS

We have also learnt about addition and subtraction of integers in class VI. let us see what we have learnt about addition and subtraction of integers in class VI.

1. If two positive (+ve) or two (–ve) integers are added, we add their values regardless of their signs and give common same sign.
   Examples: $9 + 2 = 11$; $13 + 8 = 21$; $-7 - 6 = -(7 + 6) = -13$; $-11 - 39 = -(11 + 39) = -50$
   Note that sum of two positive integers is positive and sum of two negative integers is negative.

2. For addition of a +ve and a –ve integer, we find the difference between their values and give sign of greater number.

Example: $(-7) + 15 = (15 - 7) = 8$; $(-18) + 11 = -(18 - 11) = -7$

3. (*i*) Adding a negative integer is the same as subtracting a positive integer.

Examples: $7 + (-2) = 7 - 2 = 5$; $9 + (-6) = 9 - 6 = 3$; $-3 + (-5) = -3 - 5 = -(3 + 5) = -8$

(*ii*) Subtracting a negative integer is the same as adding a positive integer.

Examples: $12 - (-3) = 12 + 3 = 15$; $-7 - (-5) = -7 + 5 = -(7 - 5) = -2$

Adding and Subtracting Integers using Number Line

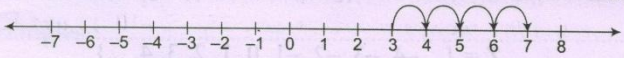
Rules:

1. When we add, we move right on the number line.
2. When we subtract, we move left on the number line.

Let us evaluate the following using number line.

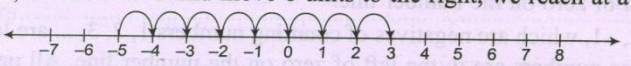
(*i*) $3 + 4$ (*ii*) $-5 + 8$ (*iii*) $-4 - 3$

(*i*) To evaluate $3 + 4$, we start at 3 and move 4 units to the right, we reach at a point 7.



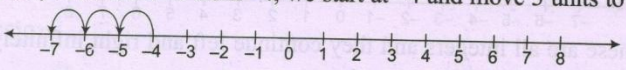
Thus, $3 + 4 = 7$

(*ii*) To evaluate $-5 + 8$, we start at -5 and move 8 units to the right, we reach at a point 3.



Thus, $-5 + 8 = 3$

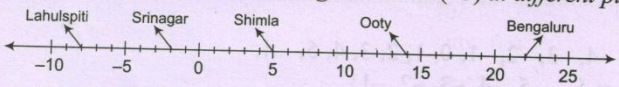
(*iii*) To evaluate $-4 - 3$ *i.e.*, to subtract -3 from -4, we start at -4 and move 3 units to the left, we reach at -7.



Thus, $-4 - 3 = -7$

Solved Examples

1. *Following number line show the temperature in degree celsius (°C) at different places on a particular day.*



(*i*) *Observe the number line and write the temperature of the places marked on it.*

(*ii*) *What is the temperature difference between the hottest and the coldest places among the above?*

(*iii*) *What is the temperature difference between Lahulspiti and Srinagar?*

(*iv*) *Can we say temperature of Srinagar and Shimla taken together is less than the temperature at Shimla? Is it also less than the temperature at Srinagar?*

Sol. (*i*) From the number line, we observe that:

Place	Temperature
Lahulspiti	–8°C
Srinagar	–2°C
Shimla	5°C
Ooty	14°C
Bengaluru	22°C

(*ii*) From the number line, we see that
temperature of the hottest place (Bengaluru) = 22°C
and temperature of the coldest place (Lahulspiti) = –8°C.
So, temperature difference between hottest and coldest places among the above is
22°C – (–8) = 22°C + 8°C = 30°C

(*iii*) Temperature of Srinagar = –2°C
Temperature of Lahulspiti = –8°C
So, temperature difference between Lahulspiti and Srinagar = –2°C – (–8°C) = –2°C + 8°C = 6°C

(*iv*) Temperature of Srinagar = –2°C
Temperature of Shimla = 5°C
So, the temperature of Srinagar and Shimla taken together = –2C° + 5°C = 3°C
As 3°C is less than 5°C, so we can say that temperature of Srinagar and Shimla taken together is less than the temperature at Shimla.
Again 3°C is greater than –2°C, so we can say that temperature of Srinagar and Shimla taken together is not less than the temperature at Srinagar.

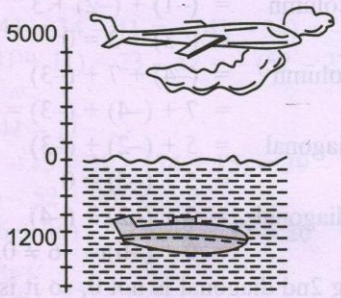
2. *In a quiz, positive marks are given for correct answers and negative marks are given for incorrect answers. If Jack's scores in five successive rounds were* 25, –5, –10, 15 *and* 10. *What was his total at the end?*

Sol. Jack's scores in five successive rounds were 25, –5, –10, 15 and 10.
So, his total score at the end = Sum of scores of all rounds
= 25 + (–5) + (–10) + 15 + 10
= (25 + 15 + 10) + (–5 – 10)
= 50 + (–15) = 50 – 15 = 35
Hence, his total score at the end = 35

3. *At Srinagar temperature was –5°C on Monday and then it dropped by 2°C on Tuesday. What was the temperature of Srinagar on Tuesday? On Wednesday it rose by 4°C. What was the temperature on this day?*

Sol. On Monday, temperature at Srinagar = –5°C.
As it dropped by 2°C on Tuesday, so temperature of Srinagar on Tuesday = –5°C – 2°C = –7°C.
Again rise in temperature at Srinagar on Wednesday
∴ Temperature at Srinagar on Wednesday = –7°C + 4°C = –3°C

4. *A plane is flying at the height of* 5000 *m above the sea level. At a particular point, it is exactly above a submarine floating* 1200 *m below the sea level. What is the vertical distance between them?*



Sol. A plane is flying at the height of 5000 m above the sea level.
Submarine is floating 1200 m below the sea level.
Vertical distance between the plane and submarine
= 5000 m – (–1200 m)
= 5000 m + 1200 m = 6200 m.

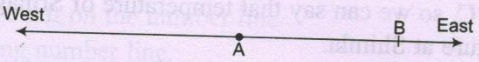
5. *Mohan deposits* ₹ 2,000 *in his account and withdraws* ₹ 1,642 *from it the next day. If withdrawal of amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Mohan's account after the withdrawal.*

Sol. If withdrawal of amount from the account is represented by a negative integer, then amount deposited which is opposite of withdrawal of amount is represented by a positive integer.

So, the balance in Mohan's account after the withdrawal

$= ₹\ 2000 + (-₹\ 1642) = ₹\ 2000 - ₹\ 1642 = ₹\ 358$

6. *Rita goes* 20 *km towards east from a point A to the point B. From B, she moves* 30 *km towards west along the same road. If the distance towards east is represented by a positive integer then, how will you represent the distance travelled towards west? By which integer will you represent her final position from A?*



Sol. As West is a direction opposite to East so distance travelled towards West by Rita will be represented by a negative integer.

Now, distance travelled by Rita from a point A to point B towards East = 20 km

She moves (–30) km towards West along the same road.

Her final position from A = 20 km to (–30) km = –(30 – 20) km = –10 km.

7. In a magic square, each row, column and diagonal have the same sum. Check which of the following is a magic square.

5	–1	–4
–5	–2	7
0	3	–3

(*i*)

1	–10	0
–4	–3	–2
–6	4	–7

(*ii*)

Sol. (*i*) Sum of integers along 1st row $= 5 + (-1) + (-4)$

$= 5 + (-5) = 5 - 5 = 0$

Sum of integers along 2nd row $= (-5) + (-2) + 7$

$= (-7) + 7 = 0$

Sum of integers along 3rd row $= 0 + 3 + (-3) = 0 + 0 = 0$

Sum of integers along 1st column $= 5 + (-5) + 0$

$= 0 + 0 = 0$

Sum of integers along 2nd column $= (-1) + (-2) + 3$

$= (-3) + 3 = 0$

Sum of integers along 3rd column $= (-4) + 7 + (-3)$

$= 7 + (-4) + (-3) = 7 + (-7) = 0$

Sum of integers along 1st diagonal $= 5 + (-2) + (-3)$

$= 5 + (-5) = 0$

Sum of integers along 2nd diagonal $= 0 + (-2) + (-4)$

$= 0 + (-6) = -6 \neq 0$

As the sum of integers along 2nd diagonal is not 0, so it is not a magic square.

(*ii*) Similar to part (*i*), in this square sum of integers along each row is –9, sum of integers along each column is –9 and sum of integer along each diagonal is –9, so it is a magic square.

8. *Verify* $a - (-b) = a + b$ *for the following values of a and b.*

(*i*) $a = 21, b = 18$ (*ii*) $a = 118, b = 125$

(*iii*) $a = 75, b = 84$ (*iv*) $a = 28, b = 11$

Sol. (*i*) Given: $a = 21, b = 18$

Now, $a - (-b) = 21 - (-18) = 21 + 18 = 39$

and $a + b = 21 + 18 = 39$

$\therefore$ $a - (-b) = a + b$ is verified [$\because$ Each side = 39]

(*ii*) Given: $a = 118, b = 125$

Now, $a - (-b) = 118 - (-125) = 118 + 125 = 243$

and $a + b = 118 + 125 = 243$

As $a - (-b) = 243$ and $a + b = 243$

So, $a - (-b) = a + b$ is verified.

(*iii*) Given: $a = 75, b = 84$

$\therefore$ $a - (-b) = 75 - (-84) = 75 + 84 = 159$

and $a + b = 75 + 84 = 159$

As $a - (-b) = 159$ and $a + b = 159$

So, $a - (-b) = a + b$ is verified.

(*iv*) Given: $a = 28, b = 11$

$\therefore$ $a - (-b) = 28 - (-11) = 28 + 11 = 39$

and $a + b = 28 + 11 = 39$

As $a - (-b) = 39$ and $a + b = 39$

So $a - (-b) = a + b$ is verified.

9. *Use the sign of >, < or = in the box to make the statements true.*

(*i*) $(-8) + (-4) \square (-8) - (-4)$ (*ii*) $(-3) + 7 - (19) \square 15 - 8 + (-9)$

(*iii*) $23 - 41 + 11 \square 23 - 41 - 11$ (*iv*) $39 + (-24) - (15) \square 36 + (-52) - (-36)$

(*v*) $-231 + 79 + 51 \square -399 + 159 + 81$

Sol. (*i*) $(-8) + (-4) = -(8 + 4) = -12$

and $(-8) - (-4) = (-8) + 4 = -(8 - 4) = -4$

As -12 is less than -4,

So, $(-8) + (-4) \leq (-8) - (-4)$

(*ii*) $(-3) + 7 - (19) = -3 + 7 - 19 = -(3 + 19) + 7$

$= -22 + 7 = -(22 - 7) = -15$

and $15 - 8 + (-9) = 15 - 8 - 9 = 15 - (8 + 9)$

$= 15 - 17 = -(17 - 15) = -2$

As -15 is less than -2,

So, $(-3) + 7 - (19) \leq 15 - 8 + (-9)$

(*iii*) $23 - 41 + 11 = (23 + 11) - 41 = 34 - 41 = -(41 - 34) = -7$

and $23 - 41 - 11 = 23 - (41 + 11) = 23 - 52 = -(52 - 23) = -29$

As -7 is greater than -29

So, $23 - 41 + 11 \geq 23 - 41 - 11$

(*iv*) $39 + (-24) - (15) = 39 - 24 - 15 = 39 - (24 + 15) = 39 - 39 = 0$

and $36 + (-52) - (-36) = 36 - 52 + 36$

$= (36 + 36) - 52 = 72 - 52 = 20$

As 0 is less 20,

So, $39 + (-24) - (15) \leq 36 + (-52) - (-36)$

(*v*) $-231 + 79 + 51 = -231 + 130 = -(231 - 130) = -101$

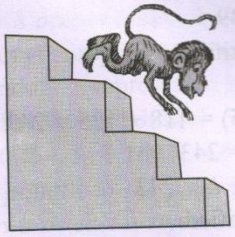
and $-399 + 159 + 81 = -399 + (159 + 81)$

$= -399 + 240 = -(399 - 240) = -159$

As -101 is less than -159

So $-231 + 79 + 51 \geq -399 + 159 + 81$

10. *A water tank has steps inside it. A monkey is sitting on the topmost step (i.e., the first step). The water level is at the ninth step.*



(*i*) *He jumps 3 steps down and then jumps back 2 steps up in every move. In how many jumps will he reach the water level?*

(*ii*) *After drinking water, he wants to go back. For this, he jumps 4 steps up and then jumps back 2 steps down in every move. In how many jumps will he reach back the top step?*

(*iii*) *If the number of steps moved down is represented by negative integers, and the number of steps moved up by positive integers, represent his moves in part (i) and (ii) by completing the following.*

(*a*) $-3 + 2 - \ldots\ldots = -8$ (*b*) $4 - 2 + \ldots\ldots = 8$.

In (a), the sum (–8) represents going down by eight steps. So, what will the sum 8 in (b) represent?

Sol. A water tank has steps inside it and a monkey is sitting at the topmost step (*i.e.*, the first step). The water level is at the ninth step.

(*i*) Let jumping one step down be represented by –1 and 1 step upward by +1.

He jumps 3 steps down and then jumps 2 steps up. He goes down by $-3 + 2 = -1$ step.

To reach water level which is at the ninth step, he has to jump as follows

$$(-3 + 2) + (-3 + 2) + (-3 + 2) + (-3 + 2) + (-3 + 2) - 3 = -8$$

So at the 8 steps down, he can drink water from the ninth step.

He takes 6 downward jumps and 5 upward jumps *i.e.*, total 11 jumps to reach the water level.

(*ii*) After drinking water, he wants to go back. For this, he jumps 4 steps up and then jumps back 2 steps down in every move.

So, $\{4 + (-2)\} + \{4 + (-2)\} + 4 = 8$

So he takes, 3 steps upward jumps and 2 downward jumps. Therefore in 5 jumps, he will reach back to the top step.

(*iii*) (*a*) $-3 + 2 - 3 + 2 - 3 + 2 - 3 + 2 - 3 + 2 - 3 = -8$

(*b*) $4 - 2 + 4 - 2 + 4 = 8$

In (*a*), the sum (–8) represents going down by eight steps

So in (*b*), sum 8 represents going up by eight steps.

11. *The sum of two integers is –16. If one of them is 12, find the other.*

Sol. The sum of two integers is –16 and one of them is 12.

Let the other integer be a, then

$$12 + a = -16$$

$$\Rightarrow \quad a = -16 + (-12) = -28$$

Hence, the other integer is –28.

12. *The sum of two integers is 72. If one of them is –18, find the other.*

Sol. Let the other integer be a. Then

$$a + (-18) = 72$$

$$\Rightarrow \quad a - 18 = 72 \quad \Rightarrow \quad a = 72 + 18 = 90.$$

Hence, the other integer is 90.

13. *The difference of an integer x and (–9) is 6. Find the value of x.*

Sol. Using the given information, we get

$x - (-9) = 6$

$\Rightarrow \quad x + 9 = 6 \Rightarrow x = 6 - 9 = -3$

Hence, $x = -3$.

14. *The difference of an integer a and (–7) is –2. Find the value of a.*

Sol. According to the question, we have

$a - (-7) = -2$

$\Rightarrow \quad a + 7 = -2 \Rightarrow a = -2 - 7 = -9$

Hence, $a = -9$

Exercise 1.1

1. Fill in the blanks:

(*i*) 0 is ..... negative nor .....

(*ii*) The numbers –1, –2, –3, –4, –5, ... are called ..... integers.

(*iii*) The numbers 1, 2, 3, 4, 5, ... are called ..... integers.

(*iv*) All negative integers are ..... than 0 and 0 is less than all ..... integers.

(*v*) The ascending order of integers –7, –1, –3, 0, –2, 8, 3, 12, is .....

(*vi*) The descending order of integers 2, –5, –1, 6, 15, –9, 8 is .....

2. State true or false for each of the following statements.

(*i*) Every positive integer have a corresponding negative integer *i.e.,* +5 and –5 are labelled as opposites.

(*ii*) When one positive and one negative integers are added, we take their difference and place the sign of the bigger integer.

(*iii*) Additive inverse of any integer *a* is $-a$ and additive inverse of $(-a)$ is *a*.

(*iv*) $(-100) - (-135) = -100 + 135 = 35$

(*v*) 20°C below normal temperature when written in the form of integer is –20°C

(*vi*) Larger number in the pair –65, 195 is –65

3. (*i*) Write all integers between –5 and 5

(*ii*) Using the number line with integer, find what is 5 less than –2.

(*iii*) Fill in the blanks by using appropriate symbol < or >

(*a*) –12 ..... 5 (*b*) 8 3

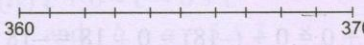
(*iv*) Write opposite of the statement: Going 20 m towards East.

(*v*) Write opposite of the statement: 10 km above the sea level.

4. Gopi deposits ₹ 4500 in his bank account in April 2016. He withdraws ₹ 1350 from it in May 2016. If withdrawal of amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Gopi's account after the withdrawal?

5. A plane is flying at the height of 4850 m above sea level. At a particular point, it is exactly above a submarine floating 1435 m below the sea level. What is the vertical distance between them.

6. Here is a part of a number line.



Put an X on the number line on the place, that is half way between 360 and 370. What is the number that should be there?

PROPERTIES OF ADDITION AND SUBTRACTION OF INTEGERS

1. Closure property

(*i*) *Integers are closed under addition.*

For any two integers *a* and *b*, $a + b$ is an integer. **Examples:** $5 + 4 = 9$; $(-8) + 2 = -6$

(*ii*) *Integers are closed under subtraction.*

For any two integers *a* and *b*, $a - b$ is an integer.

Examples: $7 - 2 = 5$; $(-18) - (-3) = -18 + 3 = -15$

2. Commutative property

(*i*) *If we change the order of integers in addition expression, the result remains the same.*

For any two integers a and b,

$$a + b = b + a.$$

Examples:

(*a*) $7 + 8 = 15$ and $8 + 7 = 15$

$\therefore \quad 7 + 8 = 8 + 7$

(*b*) $13 + (-9) = 13 - 9 = 4$ and $(-9) + 13 = -9 + 13 = 4$

$\therefore \quad 13 + (-9) = (-9) + 13$

(*c*) $(-6) + (-2) = (-8)$ and $(-2) + (-6) = (-8)$

$\therefore \quad (-6) + (-2) = (-2) + (-6)$

We see that in both the orders, the result is some.

So, *addition is commutative for integers.*

(*ii*) *Subtraction is not commutative for integers.*

For any two integers a and b, $a - b \neq b - a$

Example: $7 - (-9) = 7 + 9 = 16$

and $(-9) - 7 = -9 - 7 = -16$

$\therefore \quad 7 - (-9) \neq (-9) - 7$

3. Associative property

(*i*) *The sum of three integers does not depend upon the grouping of integers.*

i.e., the sum of three integers remains same on changing the grouping of the integers.

For any three integers a, b and c,

$$a + (b + c) = (a + b) + c$$

Example: $7 + (-8 + 6) = 7 + (-2) = 5$

and $(7 - 8) + 6 = (-1) + 6 = 5$

$\therefore \quad 7 + (-8 + 6) = (7 - 8) + 6$

(*ii*) *Subtraction is not associative for integers.*

For any three integers a, b and c,

$$a - (b - c) \neq (a - b) - c$$

Example: $8 - (9 - 4) = 8 - 5 = 3$

and $(8 - 9) - 4 = (-1) - 4 = -5$

$\therefore \quad 8 - (9 - 4) \neq (8 - 9) - 4$

4. Additive identity

Integer 0 *is an identity for integers under addition. i.e.,* when 0 is added to any integer, we get the same integer.

For any integer a, $a + 0 = 0 + a = a$

Examples: $7 + 0 = 0 + 7 = 7$; $(-18) + 0 = 0 + (-18) = 0 - 18 = -18$

5. Additive inverse

The sum of an integer and its additive inverse is 0.

For any integer a, $a + (-a) = 0$

Examples: $8 + (-8) = 8 - 8 = 0$; $-5 + (5) = -5 + 5 = 0$

Solved Examples

1. *Write down a pair of integers whose*

(*i*) *sum is* -7 (*ii*) *difference is* -10 (*iii*) *sum is* 0.

Sol. (*i*) $(-11) + 4 = -7$ or $(-9) + 2 = -7$

(*ii*) $(-19) - (-9) = -10$ or $(-26) - (-16) = -10$

(*iii*) $(-8) + 8 = 0$ or $13 + (-13) = 0$

2. (*i*) *Write a pair of negative integers whose difference gives* 8.

(*ii*) *Write a negative integer and a positive integer whose sum is* -5

(*iii*) *Write a negative integer and a positive integer whose difference is* -3.

Sol. (*i*) $-6 - (-14) = 8$ or $-17 - (-25) = 8$

(*ii*) $(-15) + 10 = -5$ or $(-8) + 3 = -5$.

(*iii*) $(-1) - 2 = -3$ or $(-2) - 1 = -3$

3. *In a quiz, team A scored* -40, 10, 0 *and team B scored* 10, 0, -40 *in three successive rounds. Which team scored more? Can we say that we can add integers in any order?*

Sol. Team A scored -40, 10, 0

Total score of team A $= (-40) + 10 + 0$

$= (-40) + 10$ [$\because$ $10 + 0 = 10$, 0 is the additive identity.]

$= -(40 - 10) = -30$

Team B scored 10, 0, -40

Total score of team B $= 10 + 0 + (-40)$

$= 10 + (-40)$ [$\because$ $10 + 0 = 10$, 0 is the additive identity]

$= 10 - 40 = -(40 - 10) = -30$

Scores of both the teams A and B are same *i.e.*, -30.

Yes, we can add integers in any order.

4. *Fill in the blanks to make the following statements true:*

(*i*) $-5 + (-8) = (-8) + (\ldots\ldots)$ (*ii*) $-53 + \ldots\ldots = -53$

(*iii*) $17 + (\ldots\ldots) = 0$ (*iv*) $[13 + (-12)] + (\ldots\ldots) = 13 + [(-12) + (-7)]$

(*v*) $(-4) + [15 + (-3)] = [-4 + 15] + \ldots\ldots$

Sol. (*i*) $-5 + (-8) = (-8) + (\mathbf{-5})$ [By commutative property of addition]

(*ii*) $-53 + \mathbf{0} = -53$ [0 is the additive identity]

(*iii*) $17 + (\mathbf{-17}) = 0$ [(-17) is the additive inverse of 17]

(*iv*) $[13 + (-12)] + (\mathbf{-7}) = 13 + [(-12) + (-7)]$ [By associative property of addition]

(*v*) $(-4) + [15 + (-3)] = [-4 + 15] + (\mathbf{-3})$ [By associative property of addition]

5. *If* $a = 3$, $b = 5$, $c = -4$, *verify*

(*i*) $a + b = b + a$ (*ii*) $a + (b + c) = (a + b) + c$

(*iii*) $(a - b) \neq (b - a)$ (*iv*) $c + 0 = c = 0 + c$

Sol. (*i*) $a + b = 3 + 5 = 8$ and $b + a = 5 + 3 = 8$

$\therefore$ $a + b = b + a$ is verified because each side = 8.

(*ii*) $a + (b + c) = 3 + \{5 + (-4)\}$

$= 3 + (5 - 4) = 3 + 1 = 4$

and $(a + b) + c = (3 + 5) + (-4) = 8 - 4 = 4$

$\therefore$ $a + (b + c) = (a + b) + c$ is verified because each side = 4.

(*iii*) $a - b = 3 - 5 = -2$ and $b - a = 5 - 3 = 2$

As $-2 \neq 2$, so $a - b \neq b - a$ is verified

(*iv*) $c + 0 = -4 + 0 = -4$, also $c = -4$

and $0 + c = 0 + (-4) = 0 - 4 = -4$

As $c + 0 = -4$, $c = -4$ and $0 + c = -4$

So, $c + 0 = c = 0 + c$ is verified.

Exercise 1.2

1. Answer each of the following questions:
   (*i*) Does the order in which two integers are added make a difference?
   (*ii*) When you are adding three integers together does it matter which two integers you add first?
   (*iii*) Does adding zero to or subtracting zero from a integer change the value of a number?
2. Identify the addition properties used in each of the following questions.
   (*i*) $7 + 13 = 13 + 7$ (*ii*) $6 + 0 = 6$
   (*iii*) $8 + (1 + 19) = (8 + 1) + 19$ (*iv*) $12 + (-12) = 0$
3. Put the brackets in each of the following additions to satisfy addition property.
   (*i*) $15 + 4 + 1 = (15 + 4) + 1$ (*ii*) $18 + (7 + 5) = 18 + 7 + 5$
4. (*i*) Give an example to show that commutative property does not exist for subtraction of integers.
   (*ii*) Give an example to show that associative property does not exist for subtraction of integers.
5. Take $a = -6$, $b = 9$, $c = 5$ and verify that
   (*i*) $a + b = b + a$ (*ii*) $a - b \neq b - a$
   (*iii*) $a + (b + c) = (a + b) + c$ (*iv*) $a - (b - c) \neq (a - b) - c$
   (*v*) $b + 0 = b$
6. Verify $a - (b - c) \neq (a - b) - c$ for $a = 9$, $b = 6$, $c = 4$
7. Match the two columns

Column I	Column II
(*i*) $17 + (-11) = (-11) + 17$	(*a*) Integers are closed under subtraction
(*ii*) $8 + 0 = 8$	(*b*) Subtraction is not associative for integers
(*iii*) $15 - 9 = 6$ (an integer)	(*c*) Commutative property of addition
(*iv*) $(6 - 7) - 5 \neq 6 - (7 - 5)$	(*d*) Addition is associative for integers.
(*v*) $(-6) + [(-7) + (-3)] = [(-6) + (-7)] + (-3)$	(*e*) 0 is the additive identity for integers

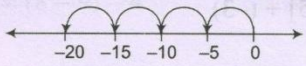
MULTIPLICATION OF INTEGERS

We know that multiplication of whole numbers is repeated addition.

Example: $7 + 7 + 7 + 7 = 4 \times 7$ ("4 times 7") $= 28$

and $9 + 9 + 9 = 3 \times 9$ ("3 times 9") $= 27$

Observe the number line given below and understand the multiplication of integers.



From the number line given above, we have

$$(-5) + (-5) + (-5) + (-5) = -20$$

We can also write

$$(-5) + (-5) + (-5) + (-5) = 4 \times (-5) \quad \text{("4 times } -5\text{")}$$

$$\therefore \quad 4(-5) = -20$$

Similarly, $(-7) + (-7) + (-7) + (-7) + (-7) = 5 \times (-7) = -35$

Rules for Multiplying Integers

1. When integers have same sign

The product of two integers, both positive or both negative (*i.e.*, like signs), is always positive. We multiply the two positive or two negative integers as whole numbers.

Examples:

(*i*) $(+3) \times (+5) = 3 \times 5 = 15$ (*ii*) $(-7) \times (-4) = 7 \times 4 = 28$